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Spectral Analyses of Wave Motions in the Middle Atmosphere

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Spectral analyses of wave motions in the middle atmosphere

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Data obtained from the selective chopper radiometer on the Nimbus 5 satellite provide a four dimensional (space and time) view of the temperature structure of the middle atmosphere. Spectral analyses of these observations enable the structure of the large scale planetary waves (zonal wavenumbers 1–6) to be deduced.

Analyses of two years of data (1973–4) reveal the important differences that exist between the wave motions in the two Hemispheres. In the Northern Hemisphere the wave motions are dominated by stationary waves, whereas in the Southern Hemisphere, eastward travelling waves are more in evidence. A detailed investigation of data during December–February 1973–4 reveals westward travelling wave 1 components with periods of 5.8, 8.3 and 12.3 days.

INTRODUCTION

In this paper we present the results of spectral analyses of stratospheric temperatures derived from the selective chopper radiometer (s.c.r.) flown on the Nimbus 5 meteorological satellite. The radiances measured by this instrument are weighted means over a vertical height range of 10–15 km (see Ellis *et al.* 1973). The three channels used in this study are B12, A1, and A2 centred at approximately 2, 50 and 100 mbar. The black body temperatures appropriate to the measured radiances are used rather than those derived from a retrieved temperature profile. The period analysed, divided into seasons, is taken from 2 years of s.c.r. data, from December 1972 to December 1974.

As the Earth rotates under an orbiting satellite, observations at a particular latitude are equally spaced in both longitude and time. (This is in contrast to the more usual meteorological measurements which are made globally at fixed times.) This relation between time and longitude allows travelling wave spectra to be obtained by a single Fourier transform of the data.

The temperature field T over a period of time corresponding to m orbits can be expanded as a Fourier sum

$$T(\phi, z, \lambda, t) = \sum_{l=-\infty}^{\infty} \sum_{n=0}^{\infty} A_{ln}(\phi, z) \exp\{\mathrm{i}(2\pi l t/m\tau - n\lambda)\}, \tag{1}$$

where ϕ is latitude, λ longitude, z height, t time, τ orbital period of satellite, n zonal wavenumber (wave), $l = n\tau\omega/2\pi$, and ω is angular frequency of the travelling wave (positive for eastward, negative for westward). The jth measurement occurs at a time $j\tau$ and a longitude $-j\tau\Omega$ (Ω = rate of rotation of the earth) so that the series of measurements (at a single latitude) are given by

$$T_j = \sum_{ln} A_{ln} \exp\{\mathrm{i}j(2\pi l/m + n\Omega\tau)\}. \tag{2}$$

It is apparent that the same series of observations, T_j, would be obtained, for instance from a wave 1 ($n = 1$) travelling eastward with a frequency of 0.5 cycle/day ($l = 0.5\ \Omega\tau$), as would be obtained from a wave 2 ($n = 2$) travelling westward at the same frequency ($l = -0.5\ \Omega\tau$).

If we assume that the waves are of planetary scale with frequencies less than 0.5 c/d then the ambiguity disappears. (This conclusion is not affected by the fact that $\Omega\tau$ is not in general an integer.) Further details are given by Chapman *et al.* (1974). The spectra show peaks clearly concentrated around frequencies of 0, 1, 2, 3...c/d which can be unambiguously assigned to the zonal mean and waves 1, 2, 3.... Peaks at the integer frequencies correspond to stationary waves, while an eastward travelling wave appears displaced to higher frequency and a westward to lower (e.g. in figure 1, wave 2 is displaced to 2.07 c/d so that the peak represents a wave travelling eastward with a frequency of 0.07 c/d, or a period of 14 days). The orbital period of the satellite restricts the highest zonal wavenumber that can be resolved to wave 6; however, in this paper we concentrate on the largest scale waves (waves 1, 2 and 3).

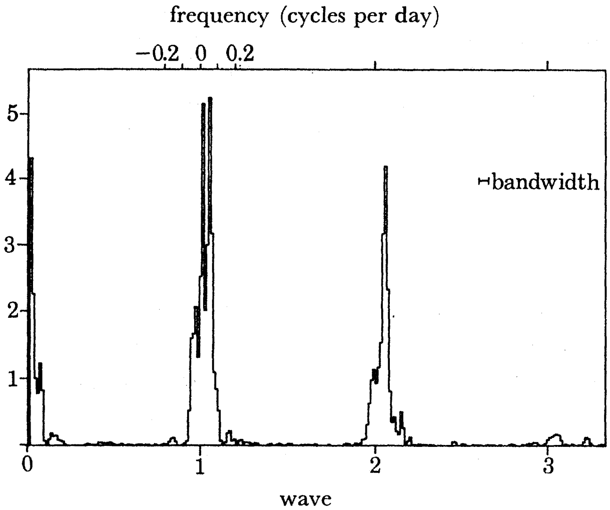

FIGURE 1. Power spectrum of temperature for latitude 60° S, channel B12 (*ca.* 2 mbar) during September–November 1973.

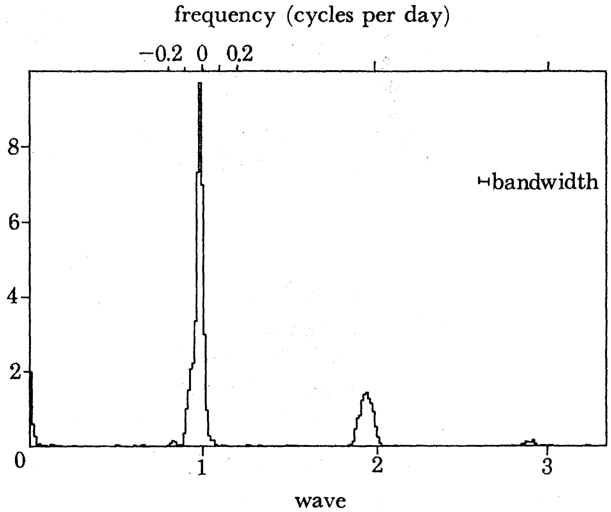

FIGURE 2. Power spectrum of temperature for latitude 60° N, channel B12 (*ca.* 2 mbar) during March–May 1974.

SEASONAL SPECTRA

The spectra described in this paper were obtained by carrying out a discrete Fourier transform of a sequence of measurements from 1024 successive orbital crossings of a particular latitude. The data, covering a period of approximately 76 days, were apodized before transforming to prevent false structure arising. This apodization provides a spectral resolution of approximately 0.03 c/d. The vertical coordinate is spectral power density (K^2 day).

The spectrum for the Southern Hemisphere at latitude 60° during spring 1973 (September–November) for channel B12 (2 mbar) is shown in figure 1. A strong wave 1 stationary peak appears together with displaced peaks representing eastward and westward travelling waves with periods close to 20 days. In contrast, wave 2 consists almost entirely of an eastward travelling component. The spectrum for channel B12 taken at latitude 60° N during spring 1974 (March–May) is shown in figure 2. The wave 1 peak is narrower than that in the Southern Hemisphere, representing a more stable stationary wave. A small westward travelling component can also be distinguished with a period around 5.8 days. Although this peak appears insignificant in this spectrum, its amplitude (*ca.* 0.4 K) is comparable to that of the 5-day wave during May 1973 at 44° N (see Rodgers 1976). The wave 2 component is westward travelling with a period around 3 weeks and it is rather broad, indicating that there is some variation of frequency over the 3-month period of analysis.

The spectra of waves 1 and 2 for each season over the 2-year period are summarized in figure 3. The spectral power (K^2) is divided into stationary waves (periods > 37 days) and westward and eastward travelling waves. This diagram is for channel B12 at latitudes 60° N and 60° S. Wave 1 consists mainly of a stationary component in the Northern Hemisphere in contrast to the Southern Hemisphere where travelling waves predominate. Wave 2 in both Hemispheres shows strong eastward travelling components. Wave amplitudes are generally greater during winter in the Northern Hemisphere and spring in the Southern Hemisphere (but note that data for winter 1973 in the Southern Hemisphere are missing). Similar results were obtained for channel A1 (*ca.* 50 mbar).

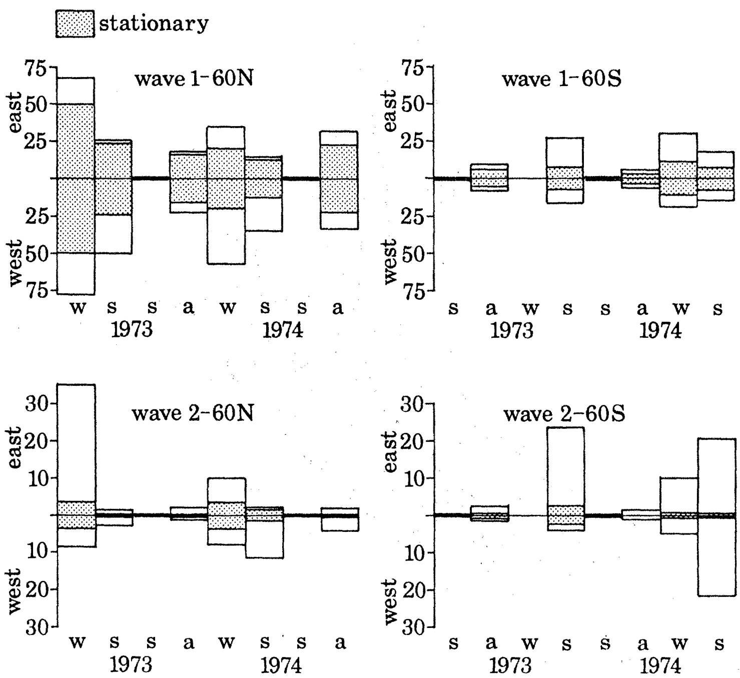

FIGURE 3. Seasonal mean power (1973–4) due to eastward and westward travelling waves and stationary waves (period < 37 days) for latitudes 60° N, 60° S waves 1 and 2, channel B12. (Note that data for winter 1973 at 60° S are missing.)

As an example of the application of this method of spectral analysis we present some preliminary results of a detailed investigation of the zonal wavenumber one structure during December–February 1973–4. Figure 4 shows a projection of the amplitudes of westward travelling waves 1 for channel A2 (100 mbar) plotted against frequency and latitude. The frequency range is from -0.03 to -0.24 c/d, and the latitude range is from 80° S to 80° N. The most obvious feature is the large amplitude stationary component in the winter Northern Hemisphere, which has been truncated in the plot. However, of more interest are the travelling waves which are present in both Hemispheres. The normal modes of oscillation of the Earth's atmosphere have been treated by a number of authors. In the simplest case of an isothermal atmosphere with no zonal winds, these modes are the solutions of the Laplace tidal equations (see, for example, Longuet-Higgins 1968). The s.c.r. will respond only to waves with a vertical wavelength of the order 10 km or greater, and we have already shown that the frequency of

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observation allows only waves of period greater than 2 days to be unambiguously resolved. This restricts our observations to the planetary scale waves, described by Hough (1898). A number of modes H_k^n are possible, depending on the zonal and meridional structure. The frequencies depend on the indices n and k and on the equivalent depth parameter which also determines the height structure. For large equivalent depths the modes become Rossby waves with stream functions the spherical harmonics P_k^n. This type of wave propagates westward.

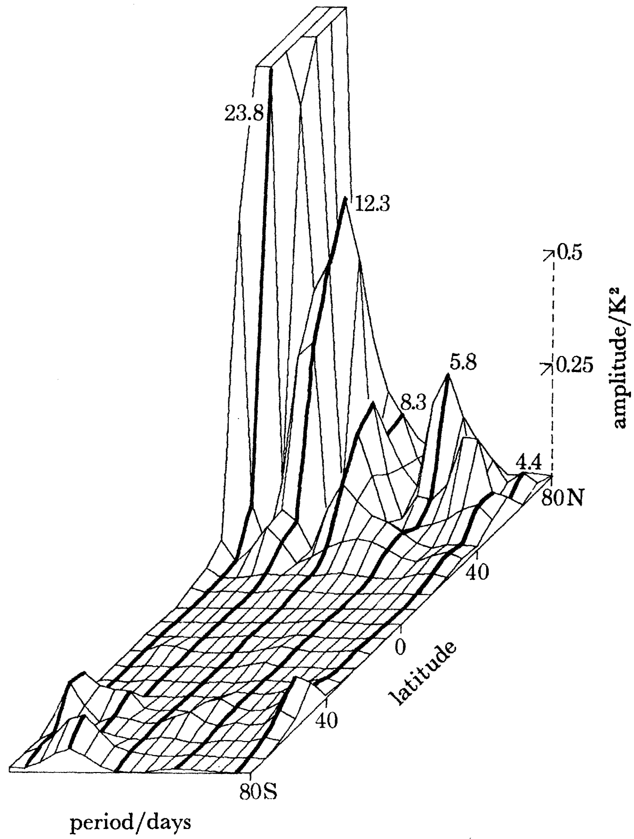

FIGURE 4. Power spectrum of the westward travelling components of wave 1 for channel A2 (*ca.* 100 mbar) during December–February 1973–4.

A number of calculations have been made with more realistic atmospheric conditions. Dickinson & Williamson (1972) derived the normal modes from a two-layer numerical model and found periods of 5, 8.3, 12.3 and 17.3 days for H_2^1 to H_5^1. (These periods are very similar to those obtained from the tidal equations with an equivalent depth of 9.5 km.) These values changed to 5.1, 10.8, 17.5 and 32.3 days when realistic zonal winds and meridional temperature gradients were introduced. The small effect on the 5-day wave has been discussed by Geisler & Dickinson (1976).

The spectra in figure 4 show strong peaks in the Northern Hemisphere at 5.4–6.2, 8.3 and 12.3 day periods. Smaller peaks corresponding to the same periods are visible in the Southern Hemisphere. These periods are very close to those calculated by Dickinson & Williamson. However, before any ambiguous assignment can be made, it will be necessary to examine the phases of the waves as functions of latitude and height. Such waves have also been observed in conventional meteorological data. Recently, Madden (1978) in an analysis extending over many years found evidence for 5- and 16-day waves in Northern Hemisphere sea level pressure and geopotential heights.

Conclusion

We have presented some results of a 2-year spectral analysis of satellite derived temperatures and confirmed the variations that occur in the spectra, both seasonally and hemispherically. In general, stationary waves tend to dominate in the Northern Hemisphere and eastward and westward travelling waves contribute similar powers to the spectra, while in the Southern Hemisphere, especially for wave 2, eastward travelling waves predominate. A preliminary study of the period from December 1973 to February 1974 revealed the presence of global westward travelling wave 1 components with periods of 5.8, 8.3 and 12.3 days (close to periods of the free modes of oscillation obtained from the solution to Laplace's tidal equations). Further analysis is being pursued in order to determine the meridional and height structure of these waves.

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